Integration by Change of Variables

Use a change of variables to compute the following integrals. Change both the variable and the limits of substitution.

a) $\displaystyle\int_0^4 \sqrt{3x+4}\,dx$

b) $\displaystyle\int_1^3 \frac{x}{x^2+1}\,dx$

c) $\displaystyle\int_0^{\pi/2} \sin^5 x \cos x\,dx$

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a) $\int_0^4 \sqrt{3x+4}\,dx$

$u = 3x+4$

$\Rightarrow du = 3\,dx$

$u = 3(4)+4 = 16$

$u = 3(0)+4 = 4$

$= \int_4^{16} \frac{1}{3}\sqrt{u}\,du$

$= \frac{1}{3}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\Big|_4^{16}$

$= \frac{2}{9}\left((16)^{\frac{3}{2}} - (4)^{\frac{3}{2}}\right)$

$= \frac{2}{9}(64-8)$

$= \frac{112}{9}$

b) $\int_1^3 \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$u = (3)^2+1 = 10$

$u = (1)^2+1 = 2$

$= \int_2^{10} \frac{1}{2}\,\frac{1}{u}\,du$

$= \frac{1}{2}\ln|u|\Big|_2^{10}$

$= \frac{1}{2}(\ln|10| - \ln|2|)$

$= \frac{1}{2}\ln 5$

c) $\int_0^{\frac{\pi}{2}} \sin^5 x \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$u = \sin\left(\frac{\pi}{2}\right) = 1$

$u = \sin 0 = 0$

$= \int_0^1 u^5\,du$

$= \frac{u^6}{6}\Big|_0^1$

$= \frac{1}{6}$